Paper Reference(s)

6675/01

Edexcel GCE

Further Pure Mathematics FP2

Advanced Level

Wednesday 18 June 2008 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP2), the paper reference (6675), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may gain no credit.

1. Show that

$$\frac{d}{dx}[\ln(\tanh x)] = 2 \operatorname{cosech} 2x, \quad x > 0.$$

(4)

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2. Find the values of x for which

$$8 \cosh x - 4 \sinh x = 13,$$

giving your answers as natural logarithms.

(6)

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3. Show that

$$\int_5^6 \frac{3+x}{\sqrt{(x^2-9)}}\, dx = 3 \ln\left(\frac{2+\sqrt{3}}{3}\right) + 3\sqrt{3} - 4.$$

(7)

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4. The curve C has equation

$$y = \operatorname{arsinh}(x^3), \qquad x \geq 0.$$

The point P on C has x-coordinate $\sqrt{2}$.

(*a*) Show that an equation of the tangent to C at P is

$$y = 2x - 2\sqrt{2} + \ln(3 + 2\sqrt{2}).$$

(5)

The tangent to C at the point Q is parallel to the tangent to C at P.

(*b*) Find the x-coordinate of Q, giving your answer to 2 decimal places.

(5)

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5. Given that

$$I_n = \int_0^{\pi} e^x \sin^n x \; dx, \qquad n \geq 0,$$

(*a*) show that, for $n \geq 2$,

$$I_n = \frac{n(n-1)}{n^2+1} I_{n-2}.$$

(8)

(*b*) Find the exact value of I_4.

(4)

6.

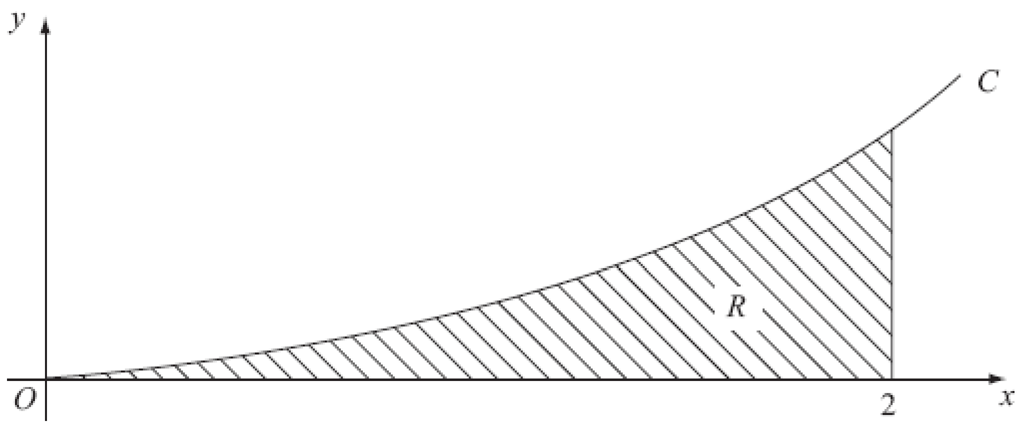

Figure 1

Figure 1 shows the curve C with equation

$$y = \frac{1}{10}\cosh x \arctan(\sinh x), \qquad x \geq 0.$$

The shaded region R is bounded by C, the x-axis and the line $x = 2$.

(*a*) Find $\int \cosh x \arctan(\sinh x) \; dx$.

(5)

(*b*) Hence show that, to 2 significant figures, the area of R is 0.34.

(2)

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7. The hyperbola H has equation

$$\frac{x^2}{16} - \frac{y^2}{9} = 1.$$

(*a*) Show that an equation for the normal to H at a point P $(4 \sec t, 3 \tan t)$ is

$$4x \sin t + 3y = 25 \tan t.$$

(6)

The point S, which lies on the positive x-axis, is a focus of H. Given that PS is parallel to the y-axis and that the y-coordinate of P is positive,

(*b*) find the values of the coordinates of P.

(5)

Given that the normal to H at this point P intersects the x-axis at the point R,

(*c*) find the area of triangle PRS.

(3)

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8. The curve C has parametric equations

$$x = 3(t + \sin t), \qquad y = 3(1 - \cos t), \qquad 0 \leq t < \pi.$$

(*a*) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \tan \dfrac{t}{2}$.

(3)

The arc length s of C is measured from the origin O.

(*b*) Show that $s = 12 \sin \dfrac{t}{2}$.

(4)

(*c*) Hence write down the intrinsic equation of C in the form $s = \mathrm{f}(\psi)$.

(1)

The point P lies on C and the arc OP of C has length L. The arc OP is rotated through 2π radians about the x-axis.

(*d*) Show that the area of the curved surface generated is given by $\dfrac{\pi L^3}{36}$.

(7)

TOTAL FOR PAPER: 75 MARKS

END

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Question number	Scheme		Marks	
1.	$\frac{\mathrm{d}}{\mathrm{d}x}(\ln(\tanh x)) = \frac{\operatorname{sech}^2 x}{\tanh x}$		M1 A1	
	$= \frac{1}{\sinh x \cosh x} = \frac{2}{\sinh 2x} = 2\operatorname{cosech} 2x$	(*)	M1 A1	(4)
				4

Notes

1M1 Any valid differentiation attempt including $\ln(e^x - e^{-x}) - \ln(e^x + e^{-x})$

1A1 c.a.o. (o.e e.g. $\frac{\cosh x}{\sinh x} - \frac{\sinh x}{\cosh x}$)

2M1 Proceeding to a hyperbolic expression in $2x$

2A1 c.s.o.

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Question number	Scheme	Marks	
2.	$8\left(\frac{e^x+e^{-x}}{2}\right)-4\left(\frac{e^x-e^{-x}}{2}\right)=13$	B1	
	$4e^x+4e^{-x}-2e^x+2e^{-x}=13$		
	$2e^{2x}-13e^x+6=0$ (or equiv.)	M1 A1	
	$(2e^x-1)(e^x-6)=0$		
	$e^x=\frac{1}{2}, \quad e^x=6$	M1 A1ft	
	$x=\ln\frac{1}{2}$ (or $-\ln 2$), $\quad x=\ln 6$	A1	(6)
			6

Notes

B1 Correctly substituting exponentials for all hyperbolics

1M1 To a three term quadratic in e^x

1A1 c.a.o. (o.e.)

2M1 Solving their equation to $e^x =$

2A1ft f.t. their equation.

3A1 c.a.o.

Question number	Scheme	Marks
3.	$\int \frac{3}{\sqrt{x^2-9}}\,dx + \int \frac{x}{\sqrt{x^2-9}}\,dx$	B1
	$= \left[3\operatorname{arcosh}\frac{x}{3} + \sqrt{x^2-9}\right]$	M1 A1 A1
	$= \left[3\ln\left(\frac{x+\sqrt{x^2-9}}{(3)}\right) + \sqrt{x^2-9}\right]_5^6$	
	$= \left(3\ln(\frac{6+\sqrt{27}}{3}) + \sqrt{27}\right) - \left(3\ln(\frac{5+4}{3}) + 4\right)$	M1 A1
	$= 3\ln\frac{6+\sqrt{27}}{9} + \sqrt{27} - 4 = 3\ln\frac{2+\sqrt{3}}{3} + 3\sqrt{3} - 4$ (*)	A1 (7)
	Notes	**7**
	B1 Correctly changing to an integrable form. **1M1** Complete attempt to integrate at least one bit. **1A1** One term correct **2A1** All correct **2DM1** Substituting limits in all. **Must have got first M1** **3A1** Correctly (no follow through) **4A1** c.s.o.	

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Question number	Scheme	Marks	
4.	(a) $\frac{dy}{dx} = \frac{3x^2}{\sqrt{1+x^6}}$, At $x = \sqrt{2}$ $\frac{dy}{dx} = \frac{6}{3} = 2$	M1 A1, A1	
	$y - \text{arsinh}\,(2\sqrt{2}) = 2(x - \sqrt{2})$	M1	
	$y = 2x - 2\sqrt{2} + \ln\left(3 + 2\sqrt{2}\right)$ (*)	A1	(5)
	(b) $\frac{3a^2}{\sqrt{1+a^6}} = 2$ $9a^4 = 4(1+a^6)$	M1 A1	
	$4a^6 - 9a^4 + 4 = 0$ $(a^2 - 2)(4a^4 - a^2 - 2) = 0$	A1	
	$a^2 = \frac{1 \pm \sqrt{1+32}}{8}$ $a = \sqrt{\frac{1+\sqrt{33}}{8}} \approx 0.92$	M1 A1	(5)
			10

Notes

(a)1M1 Attempt to differentiate need $\left(1+x^6\right)^{-\frac{1}{2}}$ at least
1A1 correct
2A1 c.a.o.
2M1 Substituting into straight line equation (linear). Must use $x = \sqrt{2}$
3A1 c.s.o.
(b)1M1 Their derivative = their gradient (condone x throughout)
2M1= A mark cao, any form
1A1 quartic cao
3M1 Solving their quartic to 'a' =
2A1 c.a.o. (a.w.r.t. 0.92 to 2dp)

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Question number	Scheme	Marks
5.	(a) $I_n = \int_0^{\pi} e^x \sin^n x\,dx = \left[e^x \sin^n x\right] - \int e^x n \sin^{n-1} x \cos x\,dx$	M1 A1
	$\left[e^x \sin^n x - ne^x \sin^{n-1} x \cos x\right] + n\int e^x(-\sin^n x + (n-1)\cos x \sin^{n-2} x \cos x)\,dx$	M1 A1
	$\left[e^x \sin^n x - ne^x \sin^{n-1} x \cos x\right]_0^{\pi} = 0$	B1
	$I_n = -n\int e^x \sin^n x\,dx + n(n-1)\int \sin^{n-2} x(1-\sin^2 x)\,dx$	M1
	$I_n = -nI_n + n(n-1)I_{n-2} - n(n-1)I_n \qquad I_n = \dfrac{n(n-1)}{n^2+1} I_{n-2}$ (*)	M1 A1 (8)
	(b) $I_4 = \dfrac{4\times 3}{17} I_2, \qquad = \dfrac{12}{17}\times\dfrac{2}{5} I_0$	M1, A1
	$I_0 = \int_0^{\pi} e^x\,dx = \left[e^x\right]_0^{\pi} = ...\qquad , \qquad I_4 = \dfrac{24}{85}\left(e^{\pi}-1\right)$	M1, A1 (4)
		12

(a)1M1 Complete attempt to use parts once in the right direction need $\sin^{n-1} x$

1A1 cao

2M1 Attempt to use parts again with sensible choice of parts, not reversing. Need to be differentiating a product.

2A1 cao

1B1 both = 0 at some point. (doesn't need to be correct, must must =0)

3DM1 I_n = expressions in $\int e^x \sin^k x\,dx$ **Depends on 2nd M**

4DM1 Expresssion in I_n and I_{n-2} to $I_n =$. **Depends on 3rd M**

3A1 c.s.o.

(b)1M1 I_4 in terms of I_2

1A1 I_4 correctly in terms of I_0 [o.e.]

2M1 $\int e^x dx$

2A1 c.a.o for I_4 .

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Question number	Scheme	Marks
6.	(a) $\int \cosh x \arctan(\sinh x)\,dx = \sinh x \arctan(\sinh x) - \int \sinh x \frac{\cosh x}{1+\sinh^2 x}\,dx$	M1 A1 A1
	$= \sinh x \arctan(\sinh x) - \frac{1}{2}\ln(1+\sinh^2 x)$ $(+C)$	M1 A1 (5)
	Or: ....$- \int \tanh x\,dx$	
	$= \sinh x \arctan(\sinh x) - \ln(\cosh x)$ $(+C)$ M1 A1	
	Alternative: Let $t = \sinh x$, $\frac{dt}{dx} = \cosh x$, $\int \arctan t\,dt = t\arctan t - \int \frac{t}{1+t^2}\,dt$ M1 A1 A1	
	$= ...... - \frac{1}{2}\ln(1+t^2)$ M1	
	$= \sinh x \arctan(\sinh x) - \frac{1}{2}\ln(1+\sinh^2 x)$ $(+C)$ (or equiv.) A1	
	(b) $\frac{1}{10}\left[\sinh x \arctan(\sinh x) - \ln(\cosh x)\right]_0^2 = ...,$ 0.34 (*)	M1, A1 (2)
		7
	(a) Alternative: Let $\tan t = \sinh x$, $\sec^2 t \frac{dt}{dx} = \cosh x$, $\int t\sec^2 t\,dt = t\tan t - \int \tan t\,dt$ M1 A1 A1 $= ...... - \ln(\sec t)$ M1 $= \sinh x \arctan(\sinh x) - \ln\sqrt{1+\sinh^2 x}$ $(+C)$ (or equiv.) A1 **Notes** **(a)1M1** Complete attempt to use parts **1A1** One term correct. **2A1** All correct. **2M1** All integration completed. Need a ln term. **3A1** c.a.o. (in x) o.e, any correct form, simplified or not **(b)1M1** Use of limits 0 and 2 and 1/10. **1A1** c.s.o.	

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Question number	Scheme	Marks	
7.	(a) $\frac{2x}{16} - \frac{2y}{9}\frac{dy}{dx} = 0$ $\left[\frac{dx}{dt} = 4\sec t\tan t,\ \frac{dy}{dt} = 3\sec^2 t\right]$	M1 A1	
	$\frac{dy}{dx} = \frac{9x}{16y} = \frac{36\sec t}{48\tan t} = \frac{3}{4\sin t}$	M1 A1	
	$y - 3\tan t = \frac{-4\sin t}{3}(x - 4\sec t)$	M1	
	$4x\sin t + 3y = 25\tan t$ (*)	A1	(6)
	(b) Using $b^2 = a^2(e^2-1)$: $ae = \sqrt{a^2+b^2} = 5$ or $e = \frac{5}{4}$	M1 A1	
	P: $4\sec t = 5$ $\cos t = \frac{4}{5}$	M1	
	Coordinates of P: $(4\sec t, 3\tan t) = \left(5, \frac{9}{4}\right)$	M1 A1	(5)
	(c) R: $x = \frac{25\tan t}{4\sin t} = \frac{125}{16}$	M1	
	Area of PRS: $\frac{1}{2}(SR \times SP) = \frac{1}{2} \times \left(\frac{125}{16} - 5\right) \times \frac{9}{4} = \frac{405}{128}\ \left(= 3\frac{21}{128}\right)$	M1 A1	(3)
		14	

Notes

(a)1M1 Differentitating
1A1 c.a.o.
2M1 $\frac{dy}{dx}$ in terms of t.
2A1 c.a.o.
3M1 Substituting gradient of **normal** into straight line equation.
3A1 c.s.o.
(b)1M1 Use of $b^2 = a^2(e^2-1)$
1A1 c.a.o. for ae or for e
2M1 Using x coordinate of focus= x coordinate of P, to get single term $f(t)$= constant. (**Allow recovery in (c)**)
3M1 Substituting into P coordinates to a number for x and for y.
2A1 c.a.o.
(c)1M1 Attempt to find x coordinate of R.
2M1 Substituting into correct template i.e. ½ x |their R_x - their H_x| x their P_y
1A1 c.a.o. 3 s.f. or better.

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8.	(a) $\dot{x} = 3 + 3\cos t \quad \dot{y} = 3\sin t$	B1
	$\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}} = \frac{\sin t}{1+\cos t} = \frac{2\sin{}^{t}\!/_{2}\cos{}^{t}\!/_{2}}{2\cos^2{}^{t}\!/_{2}} = \tan\frac{t}{2}$ (*)	M1 A1 (3)
	(b) $s = \int\sqrt{\dot{x}^2 + \dot{y}^2}\,dt = 3\sqrt{2}\int\sqrt{1+\cos t}\,dt$	M1 A1ft
	$= 6\int_0^t \cos\frac{t}{2}\,dt = 12\sin\frac{t}{2}$ (Limits or establish $C = 0$ for A1) (*)	M1 A1 (4)
	(c) $\tan\psi = \tan\frac{t}{2} \Rightarrow \psi = \frac{t}{2} \Rightarrow s = 12\sin\psi$	B1 (1)
	(d) Surface area = $\int_0^t 2\pi y\sqrt{\dot{x}^2 + \dot{y}^2}\,dt = 18\sqrt{2}\pi\int(1-\cos t)\sqrt{1+\cos t}\,dt$	M1 A1ft
	$= 72\pi\int\sin^2\frac{t}{2}\cos\frac{t}{2}\,dt$	M1
	$= \ldots\ldots\ldots\left(\frac{2}{3}\sin^3\frac{t}{2}\right)$	M1 A1
	But $\sin\frac{t}{2} = \frac{s}{12} = \frac{L}{12}$, so surface area $= \frac{144\pi}{3}\times\frac{L^3}{12^3} = \frac{\pi L^3}{36}$ (*)	M1 A1 (7)
	(a)1B1 both **1M1** Attempt at y'/x' **1A1** cso – on paper need to see half angles **(b)1M1** Attempt at arc length, integral formula **1A1** cao follow through on their x' and y' **one variable only** **2M1** Integrating **2A1** cso – on paper **(c) 1B1** cao **(d) 1M1** Attempt at Surface area, integral formula.Condone lack of 2π. **1A1** cao follow through on their x' and y' condone lack of 2π. **one variable only** **2DM1**Getting to integrable form condone lack of 2π. **Depends on previous M mark.** **3DM1**integrating condone lack of 2π. **Depends on previous M mark.** **2A1** cao **4DM1**Eliminating t to give expression in L only **Depends on previous M mark.** **3A1** cso – on paper.	

Alternative solution for 8d (from Charles)

$$S = 2\pi \int y ds$$
$$= 2\pi \int (3 - 3\cos 2\psi)(12\cos\psi) d\psi$$
$$= 2\pi \int (36\cos\psi - 36\cos\psi \cos 2\psi) d\psi$$
$$= 72\pi \int \cos\psi (1 - \cos 2\psi) d\psi$$
$$= 72\pi \int \cos\psi . 2\sin^2 \psi d\psi$$
$$= 72\pi . \frac{2}{3} \sin^3 \psi$$
$$= 48 \sin^3 \frac{t}{2}$$
$$= 48\pi \frac{L^3}{12^3}$$
$$= \frac{\pi L^3}{36}$$